Slope and Similar Triangles

Lesson 7.6
SLOPE REVIEW:

Find the slope of each line.

1. $\frac{2}{5}$
2. $-1$
3. $\frac{5}{2}$
SLOPE REVIEW:

Find the slope of the line that passes through each pair of points.

\[ A(1, 3), B(4, 7) \]
\[
\frac{7 - 3}{4 - 1} = \frac{4}{3}
\]

\[ E(4, 0), F(5, 5) \]
\[
\frac{5 - 0}{5 - 4} = 5
\]
In the figure shown, \( \triangle ABC \) and \( \triangle BDE \) are slope triangles. Slope triangles are similar.

\[ \angle BAC \cong \angle DBE \quad \text{Given} \]
\[ \angle ACB \cong \angle BED \quad \text{Given} \]
\[ \triangle ABC \sim \triangle BDE \quad \text{Angle-Angle Similarity} \]

You can use the properties of similar triangles to show the ratios of the rise to the run for each right triangle are equal.
Example

1. Write a proportion comparing the rise to the run for each of the similar slope triangles shown above. Then find the numeric value.

   Write the slope of one triangle using line segments:
   \[ \frac{AC}{BC} \]

   Write the slope of second triangle using line segments:
   \[ \frac{BE}{DE} \]

   Write the slopes as a proportion:
   \[ \frac{AC}{BC} = \frac{BE}{DE} \]

   Substitute the values of each line segment:
   \[ \frac{6}{3} = \frac{4}{2} \]

   Simplify the slope if possible: \( 2 \)
Graph each pair of similar triangles. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

\( \triangle RST \) with vertices \( R(-4, 5) \), \( S(-4, -4) \), and \( T(2, -4) \); \( \triangle UVW \) with vertices \( U(-2, 2) \), \( V(-2, -1) \), and \( W(0, -1) \)

Write the slopes as a proportion using line segments from each triangle:

\[
\frac{RS}{ST} = \frac{UV}{VW}
\]

Substitute the values of each line segment:

\[
\frac{3}{2}
\]
Graph $\triangle ACG$ with vertices $A(1, 4)$, $C(3, -2)$, and $G(1, -2)$, and $\triangle BCF$ with vertices $B(2, 1)$, $C(3, -2)$, and $F(2, -2)$. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value. (Example 1)

$$\frac{GA}{GC} = \frac{FB}{FC}, \text{ or } -\frac{3}{1} \text{ or } -3$$
Got It? Do this problem to find out.

Graph \( \triangle MNO \) with vertices \( M(3, 1) \), \( N(1, 0) \), and \( O(3, 0) \), and \( \triangle PQR \) with vertices \( P(5, 2) \), \( Q(-1, -1) \), and \( R(5, -1) \). Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

\[
\frac{MO}{NO} = \frac{PR}{QR} \quad \text{or} \quad \frac{1}{2}
\]
Graph each pair of similar triangles. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

$\triangle QRP$ with vertices $Q(-5, 1), R(-1, 3)$, and $P(-1, 1)$; $\triangle RKJ$ with vertices $R(-1, 3), K(5, 6)$, and $J(5, 3)$.

$$\frac{RP}{PQ} = \frac{KJ}{JR} \text{ or } \frac{1}{2}$$
The ratio of the rise to the run of two slope triangles formed by a line is equal to the slope of the line.

**Larger Triangle**

\[
\frac{\text{rise}}{\text{run}} = \frac{6}{-3} = -2
\]

**Smaller Triangle**

\[
\frac{\text{rise}}{\text{run}} = \frac{2}{-1} = -2
\]

Slope: \(\frac{-2}{1}\), or \(-2\)
Example

The pitch of a roof refers to the slope of the roof line. Choose two points on the roof and find the pitch of the roof shown. Then verify that the pitch is the same by choosing a different set of points.

Formula for slope

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Use the points S and R.

(\(x_1, y_1\)) = (8, 6)

\[ m = \frac{8 - 6}{12 - 8} \]

(\(x_2, y_2\)) = (12, 8)

\[ m = \frac{2}{4} \text{ or } \frac{1}{2} \]

The pitch of the roof is \(\frac{1}{2}\). Verify that the pitch is the same using two other points.
**Example**

The pitch of a roof refers to the slope of the roof line. Choose two points on the roof and find the pitch of the roof shown. Then verify that the pitch is the same by choosing a different set of points.

![Graph showing the roof pitch calculation](image)

**Formula for slope**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

**Use the points** \( U \) and \( T \).

\[ (x_1, y_1) = (2, 3) \]

\[ m = \frac{2 - 3}{0 - 2} \]

\[ m = \frac{-1}{-2} \text{ or } \frac{1}{2} \]

\[ (x_2, y_2) = (0, 2) \]
The plans for a teeter-totter are shown at the right. Using points G and L, find the slope of the teeter-totter. Then verify that the slope is the same at a different location by choosing a different set of points.

**Formula for slope:**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

**Use the points G and L:**

\[(x_1,y_1) \rightarrow (1,1)\]
\[(x_2,y_2) \rightarrow (7,3)\]

\[ m = \frac{3}{6} = \frac{1}{2} \]

**Use the points H and K:**

\[(x_1,y_1) \rightarrow (4,2)\]
\[(x_2,y_2) \rightarrow (10,4)\]

\[ m = \frac{4 - 2}{10 - 4} = \frac{1}{3} \]
Use the points X and Z:

\[
\frac{4 - 0}{7 - 1} = \frac{4}{6} = \frac{2}{3}
\]

Use the points W and Y:

\[
\frac{6 - 2}{10 - 4} = \frac{4}{6} = \frac{2}{3}
\]
Use a graph to find the missing coordinates for point $Z$ if $\triangle MNP \sim \triangle XYZ$.

$M(5, 0), N(5, -3), P(2, -3), X(7, 2), Y(1, 2)$

$Z(1, -4)$
Homework:

Pg. 565 - 568

# 1-6 (all)
# 10-26 (evens)