Polygons and Angles

Lesson 5-4
Vocabulary:

**Polygon** - A simple closed figure formed by three or more line segments. (The segments only intersect at their endpoints.)

**Equiangular** - all angles have the same measure

**Equilateral** - all sides are the same length

**Regular Polygon** - A polygon that is equilateral and equiangular.
The sum of the measures of the interior angles of a polygon is \((n - 2)180\), where \(n\) represents the number of sides.

\[
S = (n - 2)180
\]

You can use the sum of the angle measures of a triangle to find the sum of the interior angle measures of various polygons.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Sketch of Figure</th>
<th>Number of Triangles</th>
<th>Sum of Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><img src="triangle.png" alt="Triangle" /></td>
<td>1</td>
<td>(1(180^\circ) = 180^\circ)</td>
</tr>
<tr>
<td>4</td>
<td><img src="quadrilateral.png" alt="Quadrilateral" /></td>
<td>2</td>
<td>(2(180^\circ) = 360^\circ)</td>
</tr>
<tr>
<td>5</td>
<td><img src="pentagon.png" alt="Pentagon" /></td>
<td>3</td>
<td>(3(180^\circ) = 540^\circ)</td>
</tr>
<tr>
<td>6</td>
<td><img src="hexagon.png" alt="Hexagon" /></td>
<td>4</td>
<td>(4(180^\circ) = 720^\circ)</td>
</tr>
</tbody>
</table>
Names of Polygons:

Polygon Names

3 Sides = Triangle
4 Sides = Quadrangle
5 Sides = Pentagon
6 Sides = Hexagon
7 Sides = Heptagon
8 Sides = Octagon
9 Sides = Nonagon
10 Sides = Decagon
11 Sides = Undecagon
12 Sides = Dodecagon
1. Find the sum of the measures of the interior angles of a decagon.

Write the equation.

A decagon has 10 sides.
Replace \( n \) with 10.

Simplify.

\[ S = (n - 2)180 \]

\[ S = (10 - 2)180 \]

\[ S = (8)180 \]

\[ S = 1,440^\circ \]
2. Find the sum of the measures of the interior angles of a 13-gon.

Write the equation.  \[ S = (n - 2)180 \]

Replace \( n \) with 13.  \[ S = (13 - 2)180 \]

Simplify.  \[ S = (11)180 \]

\[ S = 1,980^\circ \]
## Find the sum of the interior angle measures of each polygon.

<table>
<thead>
<tr>
<th></th>
<th>a. hexagon</th>
<th>b. octagon</th>
<th>c. 15-gon</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$S = (n - 2)180$</td>
<td>$S = (n - 2)180$</td>
<td>$S = (n - 2)180$</td>
</tr>
<tr>
<td></td>
<td>$S = (6 - 2)180$</td>
<td>$S = (8 - 2)180$</td>
<td>$S = (15 - 2)180$</td>
</tr>
<tr>
<td></td>
<td>$S = (4)180$</td>
<td>$S = (6)180$</td>
<td>$S = (13)180$</td>
</tr>
<tr>
<td></td>
<td><strong>720°</strong></td>
<td><strong>1,080°</strong></td>
<td><strong>2,340°</strong></td>
</tr>
</tbody>
</table>
3. Each chamber of a bee honeycomb is a regular hexagon. Find the measure of an interior angle of a regular hexagon.

**Step 1** Find the sum of the measures of the angles.

\[ S = (n - 2)180 \]
\[ S = (6 - 2)180 \]
\[ S = (4)180 \]
\[ 720^\circ \]

**Step 2** Divide your answer from Step 1 by the number of interior angles.

\[ 720^\circ \div 6 = 120^\circ \]
4. A designer is creating a new logo for a bank. The logo consists of a regular pentagon. Find the measure of an interior angle of the regular pentagon.

**Step 1** Find the sum of the measures of the angles.

\[ S = (n - 2)180 \]

\[ S = (5 - 2)180 \]

\[ S = (3)180 \]

\[ 540^\circ \]

**Step 2** Divide your answer from Step 1 by the number of interior angles.

\[ 540^\circ \div 5 = 108^\circ \]
Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

**d. octagon**

\[
S = (8 - 2)180 = 1,080°
\]

\[
1080° ÷ 8 = 135°
\]

**e. heptagon**

\[
S = (7 - 2)180 = 900°
\]

\[
900° ÷ 7 = 128.6°
\]

**f. 20-gon**

\[
S = (20 - 2)180 = 3,240°
\]

\[
3,240° ÷ 20 = 162°
\]
Exterior Angles of a Polygon

In a polygon, the sum of the measures of the exterior angles, one at each vertex, is $360^\circ$.

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360^\circ$

Regardless of the number of sides in a polygon, the sum of the exterior angle measures is equal to $360^\circ$. 
5. Find the measure of an exterior angle in a regular hexagon.

Let $x$ represent the measure of each exterior angle.

Write an equation.

A hexagon has 6 sides and exterior angles have a sum of 360.

$$6x = 360$$

Solve using the Division Property of Equality.

$$\frac{6x}{6} = \frac{360}{6}$$

$$x = 60^\circ$$
6. Find the measure of an exterior angle in a regular 30-gon.

Let \( x \) represent the measure of each exterior angle.

Write an equation.
A 30-gon has 30 sides and exterior angles have a sum of 360.

\[
30x = 360
\]

Solve using the Division Property of Equality.

\[
\frac{30x}{30} = \frac{360}{30}
\]

\[
x = 12
\]
Find the measure of an exterior angle of each regular polygon.

**g. triangle**

\[
3x = 360 \\
\frac{3x}{3} = \frac{360}{3} \\
x = 120
\]

\[120°\]

**h. quadrilateral**

\[
4x = 360 \\
\frac{4x}{4} = \frac{360}{4} \\
x = 90
\]

\[90°\]

**i. octagon**

\[
8x = 360 \\
\frac{8x}{8} = \frac{360}{8} \\
x = 45
\]

\[45°\]
Homework:

Pg. 401 - 404

# 1-35 (odds)