Proportional and Non-proportional Relationships
Lesson 1.4

Vocabulary:
Proportional - 2 ratios with a constant rate (same)
Non-proportional - 2 ratios with a rate that is not constant (different)
Equivalent Ratios - Ratios that have the same value

Example

1. Andrew earns $18 per hour for mowing lawns. Is the amount of money he earns proportional to the number of hours he spends mowing? Explain.

<table>
<thead>
<tr>
<th>Earnings ($)</th>
<th>18</th>
<th>36</th>
<th>54</th>
<th>72</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

a) Find the unit rate for each relationship:

\[
\frac{\text{amount earned}}{\text{number of hours}} \rightarrow \frac{18}{1} \text{ or } 18
\]

\[
\frac{36}{2} \text{ or } 18
\]

\[
\frac{54}{3} \text{ or } 18
\]

\[
\frac{72}{4} \text{ or } 18
\]

b) Are all the unit rates the same? Explain.
(Yes – proportional or No – non-proportional)

Yes, the dollar to hour ratios are all equal to 18.
2. Uptown Tickets charges $7 per baseball game ticket plus a $3 processing fee per order. Is the cost of an order proportional to the number of tickets ordered? Explain.

<table>
<thead>
<tr>
<th>Cost ($)</th>
<th>10</th>
<th>17</th>
<th>24</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tickets Ordered</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

a) Find the unit rate for each relationship:

\[
\frac{\text{cost of order}}{\text{tickets ordered}} \rightarrow \frac{10}{1} \text{ or } 10
\]
\[
\frac{17}{2} \text{ or } \frac{17 - 2}{2} = 8.5
\]
\[
\frac{24}{3} \text{ or } 8
\]
\[
\frac{31}{4} \text{ or } \frac{7.75}{1} = 7.75
\]

b) Are all the unit rates the same? Explain.
(Yes – proportional or No – non-proportional)

No, the cost to tickets ordered ratios are not all equal.
3. You can use the recipe shown to make a fruit punch. Is the amount of sugar used proportional to the amount of mix used? Explain.

<table>
<thead>
<tr>
<th>Cups of Sugar</th>
<th>$\frac{1}{2}$</th>
<th>1</th>
<th>$1\frac{1}{2}$</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Envelopes of Mix</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

a) Find the unit rate for each relationship:

\[
\frac{\text{cups of sugar}}{\text{envelopes of mix}} \rightarrow \frac{\frac{1}{2}}{1} = \sqrt{\frac{1}{2}} (0.5)
\]

\[
\frac{\frac{1}{2}}{} = (0.5)
\]

\[
\frac{\frac{\frac{11}{2}}{3}}{} \rightarrow \frac{3}{2} \div \frac{3}{1} = \frac{3}{2} \div \frac{3}{1} = \sqrt{\frac{1}{2}} (0.5)
\]

\[
\frac{2}{4} \div 2 = \sqrt{\frac{1}{2}} (0.5)
\]

b) Are all the unit rates the same? Explain.

(Yes – proportional or No – non-proportional)

Yes, the cups of sugar to envelopes of mix ratios are all equal 0.5.
4. At the beginning of the year, Isabel had $120 in the bank. Each week, she deposits another $20. Is her account balance proportional to the number of weeks of deposits? Use the table below. Explain your reasoning.

<table>
<thead>
<tr>
<th>Time (wk)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance ($)</td>
<td>140</td>
<td>160</td>
<td>180</td>
</tr>
</tbody>
</table>

\[
(120 + 20) \quad (140 + 20) \quad (160 + 20)
\]

a) Find the unit rate for each relationship:

\[ \frac{140}{1} = \boxed{140} \]

\[ \frac{160}{2} = 160 \div 2 = \boxed{80} \]

\[ \frac{180}{3} = 180 \div 3 = \boxed{60} \]

b) Are all the unit rates the same? Explain.

No, the balance per week ratios are not all equal.
5. The Vista Marina rents boats for $25 per hour. In addition to the rental fee, there is a $12 charge for fuel. Is the number of hours you can rent the boat proportional to the total cost? Explain.  

<table>
<thead>
<tr>
<th>Rental Time (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>37</td>
<td>62</td>
<td>87</td>
</tr>
</tbody>
</table>

\( \text{(12+25)} \), \( \text{(37+25)} \), \( \text{(62+25)} \)

a) Find the unit rate for each relationship:

\[ \frac{\$37}{1} = \boxed{37} \]

\[ \frac{87}{3} = \frac{87 \div 3}{3} = \boxed{29} \]

\[ \frac{62}{2} = \frac{62 \div 2}{2} = \boxed{31} \]

b) Are all the unit rates the same? Explain.

No, the rental cost to hours ratios are not all equal.

6. A vine grows 7.5 feet every 5 days. Is the length of the vine on the last day proportional to the number of days of growth?

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (ft)</td>
<td>7.5</td>
<td>15</td>
<td>22.5</td>
<td>30</td>
</tr>
</tbody>
</table>

\( \text{(7.5+7.5)} \), \( \text{(15+7.5)} \), \( \text{(22.5+7.5)} \)

a) Find the unit rate for each relationship:

\[ \frac{\text{ft}}{\text{day}} \to \frac{2.5}{5} = \frac{7.5 \div 5}{5} = \boxed{1.5} \]
\[ \frac{22.5}{15} = \frac{22.5 \div 15}{15} = \boxed{1.5} \]
\[ \frac{15}{10} = \frac{15 \div 10}{10} = \boxed{1.5} \]
\[ \frac{30}{20} = \frac{30 \div 20}{20} = \boxed{1.5} \]

b) Are all the unit rates the same? Explain.

Yes, the feet to days ratios are all equal to 1.5.
Example

7. The tables shown represent the number of pages Martin and Gabriel read over time. Which situation represents a proportional relationship between the time spent reading and the number of pages read? Explain.

<table>
<thead>
<tr>
<th>Pages Martin Read</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (min)</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pages Gabriel Read</th>
<th>3</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (min)</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

a) Find the unit rate for each relationship for both tables:

**Martin**

\[
\begin{align*}
\frac{2}{5} & = 2 \div 5 = 0.4 \\
\frac{4}{10} & = 2 \div 5 = 0.4 \\
\frac{6}{15} & = 3 \div 5 = 0.4
\end{align*}
\]

**Gabriel**

\[
\begin{align*}
\frac{3}{5} & = 3 \div 5 = 0.6 \\
\frac{4}{10} & = 2 \div 5 = 0.4 \\
\frac{7}{15} & = 0.46
\end{align*}
\]

b) In which table are all the unit rates the same? Explain.

Martin's pages to minutes ratios are proportional because they are all equal to \(\frac{2}{5}\).
8. Which situation represents a proportional relationship between the hours worked and amount earned for Matt and Jane? Explain. (Example 4)

<table>
<thead>
<tr>
<th>Matt's Earnings ($)</th>
<th>12</th>
<th>20</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jane's Earnings ($)</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

a) Find the unit rate for each relationship for both tables:

Matt

Jane

b) In which table are all the unit rates the same? Explain.